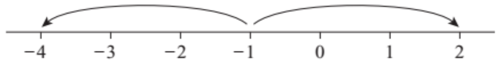
Algebra

Adding and subtracting terms

Just as you would use a number line to add and subtract integers:

$1-3=-4 \qquad -1+3=2$

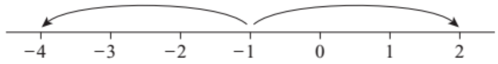
$1+(-3)=-4 \qquad -1-(-3)=2$



In the same way you use a number line to add and subtract algebraic terms:

$-x-3x=-4x \qquad -x+3x=2x$

$-x+(-3x)=-4x \qquad -x-(-3x)=2x$

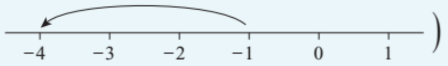


Examples

1. $m-(-4m)=m+4m=5m$
(subtracting a negative is the same as adding)

2. $-x-3x=-4x$
(compare this with:

$-1-3=-4$



)

3. $2n^2+n-5n^2=-3n^2+n$

(2 lots of n^2 minus 5 lots of n^2; but note that an n^2 term and an n term cannot be added together)

TOP TIP

1. Simplify where possible:

(a) $3pq+5pq$ (b) $7xy+2yz$
(c) $km+6mk$ (d) $12cd^2-5cd^2$
(e) $13rs^2-8r^2s$ (f) $7ax^2+3bx^2$
(g) $4v^2w+3wv^2$ (h) $9p^2q^2-q^2p^2$
(i) $3y^2-2y^2+7y-y$ (j) $r^3+2r^3-8r+5r$
(k) $b^2+3b^2-ab-2ab$ (l) $r^3+2r^3-8r+5r$
(m) $ab+3xy+5ab$ (n) $pq-kl+7pq$
(o) $8xy^2+xy-6xy^2$ (p) $9vw^2-7v^3w-vw^2$
(q) $13cd-20cd+dc$ (r) $a^3+b^3-a^3b^3$
(s) $2x^3+3x^2-x^3$ (t) $5x^2+x^3-9x^2$

2. Simplify:

(a) x^2+10-x^2+10 (b) p^2+p+p^2+2p
(c) $m^2+m+1-m^2-2m-3$ (d) $-3b^2+2b+4+b^2-2b+4$
(e) $a-2c+3b-2b+a+5c$ (f) $-p-r-q+7p-4q-r$
(g) $4x^2-5xy-2xy-x^2$ (h) $9s^2-5r^2-8r^2s^2-8s^2$
(i) $2.7k-1.3j+3.3k+0.9j$ (j) $\frac{1}{4}t^2-s+\frac{1}{2}+\frac{1}{2}t^2-\frac{1}{2}s+\frac{1}{4}$
(k) $4x^2-8y^2-3xy+5xy-2y^2$
(l) $ab+a^2b-9ab+5a^2b+b^2a$

Removing one pair of brackets

$3(m-4)$ means 'three lots of $m-4$':

$m-4 \quad m-4 \quad m-4$

$= 3m - 12$

There are: 'three lots of m' and 'three lots of -4' giving $3m$ and -12

Here you should think: $3(m-4) = 3m - 12$

In general $a(b+c) = ab + ac$ and $a(b-c) = ab - ac$

Also $a(b+c+d) = ab + ac + ad$

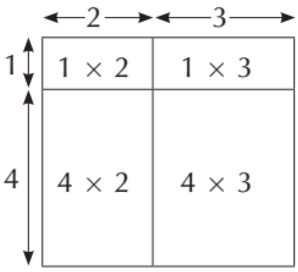
TOP TIP

When multiplying use the positive/negative rules, e.g. neg × neg = pos.

Examples

1. $5(x+3) = 5x + 15$
2. $-(n-n^2) = -n + n^2$ (think $-1(-n\ -n^2)$)
3. $-2(x-3) = -2x + 6$
4. $y(4y-5) = 4y^2 - 5y$
5. $a(2b+a) = 2ab + 2a^2$
6. $6x(3x-2) = 18x^2 - 12x$

Removing two pairs of brackets



This square has area:
5×5
$= (1 + 4)\ (2 + 3)$
or, adding individual rectangles:

$1 \times 2 + 1 \times 3 + 4 \times 2 + 4 \times 3$

First numbers in each bracket	Outside two numbers	Inside two numbers	Last numbers in each bracket

The rule is: **F** **O** **I** **L**

Which means: **F**irsts, **O**utsides, **I**nsides, **L**asts

Here is the pattern in general:

	$(a+b)\quad(c+d)$	
Firsts:	$(a+b)\quad(c+d)$	ac
Outsides:	$(a+b)\quad(c+d)$	ad
Insides:	$(a+b)\quad(c+d)$	bc
Lasts:	$(a+b)\quad(c+d)$	bd

giving:
$ac + ad + bc + bd$

TOP TIP

The FOIL method for brackets will help you to also factorise quadratics.

Example

To multiply out $(2y - 3)(y - 5)$ you should think like this:

$(2y\ -3)(y\ -5)$

F	O	I	L
$(2y)(y)$	$(2y)(-5)$	$(-3)(y)$	$(-3)(-5)$
multiply	multiply	multiply	multiply
$2y^2$	$-10y$	$-3y$	$+15$

$$\underbrace{-10y \qquad -3y}_{-13y}$$

giving $2y^2 - 13y + 15$ (notice the Outsides and Insides usually combine).

Removing squared brackets

TOP TIP

$(2x - 3)^2$ $(5x - 4)^2$ When squaring out terms like these the constant term is always POSITIVE.

Remember that m^2 means $m \times m$.

So, for example:

$(3y + 2)^2$ means $(3y + 2)(3y + 2)$

$(b - a)^2$ means $(b - a)(b - a)$

$(5 - x)^2$ means $(5 - x)(5 - x)$

and then you use the FOIL rule to get rid of the two pairs of brackets:

$$9y^2 + 6y + 6y + 4 = 9y^2 + 12y + 4$$

$$b^2 - ab - ab + a^2 = b^2 - 2ab + a^2$$

$$25 - 5x - 5x + x^2 = 25 - 10x + x^2$$

Example

Multiply out the brackets and then simplify: $(3x - 2)^2 - (3x + 2)^2$

Solution

$$(3x - 2)(3x - 2) - (3x + 2)(3x + 2)$$

$$= 9x^2 - 6x - 6x + 4 - (9x^2 + 6x + 6x + 4)^*$$

$$= 9x^2 - 12x + 4 - (9x^2 + 12x + 4)$$

$$= 9x^2 - 12x + 4 - 9x^2 - 12x - 4$$

$$= -24x$$

*Note: Brackets are needed because the term $(3x + 2)(3x + 2)$ is being subtracted.

Removing brackets containing more than two terms

For expressions such as:

$$(a + b)(c + d + e)$$

FOIL does not work.

Here is the pattern to use:

$(a + b)(c + d + e)$ giving $ac + ad + ae$

then $(a + b)(c + d + e)$ giving $bc + bd + be$.

There are a total of six multiplications:

$$(a + b)(c + d + e) = ac + ad + ae + bc + bd + be$$

Example

Expand $(5m - 2)(m^2 - 2m + 3)$

Solution

First multiply by $5m$

$(5m - 2)(m^2 - 2m + 3)$ giving $5m^3 - 10m^2 + 15m$

Now multiply by -2

$(5m - 2)(m^2 - 2m + 3)$ giving $-2m^2 + 4m - 6$

So $(5m - 2)(m^2 - 2m + 3)$

$= 5m^3 - 10m^2 + 15m$ (from $5m$)

$\quad -2m^2 + 4m - 6$ (from -2)

$= 5m^3 - 12m^2 + 19m - 6$ (combining 'like' terms)

Multiplying and dividing algebraic expressions

1. Simplify:

(a) $7a \times 3a^2b$ (b) $-8xy \times 4xy$
(c) $ab \times pq$ (d) $mn^2 \times m^2n$
(e) $-k \times (-km^2)$ (f) $cd \times (-d^2)$
(g) $\frac{1}{a} \times a$ (h) $r^2 \times \frac{p}{r}$
(i) $\frac{v}{w^2} \times \frac{w}{v}$ (j) $(2a)^2 \times 3a$
(k) $5e \times (5f)^2$ (l) $(-g)^2 \times 7fg$
(m) $(7x)^2 \times (-x)^2$ (n) $-4p \times (pqg)^2$
(o) $\left(\frac{1}{a}\right)^3 \times a^2$ (p) $\frac{3}{(2s)^2} \times 8rs$
(q) $\frac{1}{x^3} \times x^3y$ (r) $\frac{a}{b^2} \times \frac{b}{a^2}$

2. Simplify:

(a) $6ab \div 3a$ (b) $15xy \div 5y$
(c) $36cd \div 9cd$ (d) $7pq^2 \div pq$
(e) $20rs^2 \div 4s^2$ (f) $54f^2g \div 9f$
(g) $12ab^2 \div 4a^2b$ (h) $16uv \div 8uv^2$
(i) $19ab \div ab^2$ (j) $3m^2n \div 18mn$
(k) $7gh^2 \div 14h^3$ (l) $28k^2l^2 \div 7k^2l$
(m) $3tu \div 39t^2u^2$ (n) $33vw^3 \div 11vw$

3. Expand:

(a) $3x(5x+2)$ (b) $7y(y^2-z)$
(c) $-3a^2(2-5a^2)$ (d) $-(9p-q)$
(e) $12r(s^2+r)$ (f) $-w^2(vw+1)$
(g) $(13x-5y)3y$ (h) $(a^2+b^2)abc$
(i) $(-5d^3-e^2)e$ (j) $f^2g(3g-2f)$
(k) $(m^2+5n)km$ (l) $-8t^3(u-tu)$

4. Simplify:

(a) $\frac{27x+18}{3}$ (b) $\frac{35x^2-5y}{5}$
(c) $\frac{x^2-5x}{x}$ (d) $\frac{12ab+a^2}{4a}$
(e) $\frac{48p^2q-16pq}{8p}$ (f) $\frac{5r^2s^2+7rs^2}{rs}$
(g) $\frac{pqr+p^2qr^2}{pqr}$ (h) $\frac{14x^3y^2-28xy+7x^2y}{7xy}$

Preparation for Higher

Expanding brackets

1. Expand:

(a) $(x+3)(x+4)$ (b) $(4-y)(6-y)$ (c) $(p-2)(p-13)$
(d) $(r+5)(r-6)$ (e) $(4-s)(8+s)$ (f) $(2t+5)(t-7)$
(g) $(3v-1)(7v+4)$ (h) $(2-9w)(4-3w)$ (i) $2(x+5)(x+7)$
(j) $5(a+2)(a-9)$ (k) $7(1-t)(4-t)$ (l) $-4(m+2)(m-10)$

2. Expand:

(a) $(x+2)^2$ (b) $(3y-5)^2$ (c) $(7+2a)^2$
(d) $(9-4q)^2$ (e) $(x+2)(x-2)$ (f) $(3k-1)(3k+1)$
(g) $(20+7q)(20-7q)$ (h) $3(x+1)(x-1)$ (i) $5(a-2)(a+2)$
(j) $10(3+p)(3-p)$ (k) $6(m-n)(m+n)$ (l) $2(3k+4)(3k-4)$
(m) $(b+5)^2$ (n) $(d-6)^2$ (o) $(8-k)^2$
(p) $(10+p)^2$ (q) $(9+2z)^2$ (r) $(7-5v)^2$
(s) $(3p-q)^2$ (t) $(5m+2n)^2$ (u) $(a+b)^2$
(v) $(g-7h)^2$ (w) $(5x-8y)^2$ (x) $(6v+9w)^2$

3. Expand and simplify:

(a) $x(2x+4)+3x-5(x^2-6)$ (b) $x^2(5x-7)+4x(3x+2)-x^2$
(c) $8x(5x-6)-3(x+4)^2$ (d) $y^2(y+5)-7y-(y-5)^2$
(e) $9(a-6)^2-a(2a+3)^2$ (f) $25p^2-(5-p)^2+10p$
(g) $4f(f-5)^2+20f-2(3f-1)^2$
(h) $t^2(9t-1)-9(t+1)^2+10t^2$
(i) $-6s(7+s)^2+(2s-6)^2$
(j) $(8-2w)^2-60+10w-(w-4)^2$

4. Expand and simplify:

(a) $(x+1)(x^2-2x+3)$ (b) $(2y+4)(y^2+5y-6)$
(c) $(5a-2)(3a^2-7a+4)$ (d) $(b^2+5b-2)(6b-1)$
(e) $(7p^2-8p-9)(3p+6)$ (f) $(1-q)(5-2q+q^2)$
(g) $(8+t)(8-3t+3t^2)$ (h) $(6-4s-2s^2)(9-5s)$
(i) $(2+x)(3x^2+5x+1)$ (j) $(6-d)(7d^2+2d-8)$
(k) $(f+9)(5-7f+4f^2)$ (l) $(2-4h-3h^2)(6h+9)$
(m) $2(w+5)(w^2-3w+1)$ (n) $3(z^2+7z-2)(2z-1)$
(o) $5(2-a)(3a^2+a-2)$ (p) $4(1+b)(7-2b-b^2)$

5. Expand and simplify:

(a) $(k+4)^3$ (b) $(3r-5)^3$ (c) $(p-1)^3$ (d) $(4+2q)^3$
(e) $(a+2)^3$ (f) $(b-1)^3$ (g) $(2d+3)^3$ (h) $(5e-2)^3$
(i) $(1+4x)^3$

Factorising quadratic expressions

You know that if $(x-4)(x+3)=x^2-x-12$ you expand the pair of brackets using FOIL.

$x^2 \quad + \quad 3x \quad - \quad 4x \quad - \quad 12$

F O I L

Reversing this gives $x^2-x-12=(x-4)(x+3)$

(? ? ?)(? ? ?)

You factorise the quadratic expression by reversing the FOIL expansion...But how is this done?

Vital observation:

The **middle term** $-x$ is a combination of the Outsides $(+3x)$ and Insides $(-4x)$.

Example Let's try to factorise x^2+5x-6

Step 1 List all the possibilities for the Firsts and Lasts:

$(x \quad 2)(x \quad 3)$

$(x \quad 1)(x \quad 6)$

The Firsts multiply to give x^2 so $x \times x$.

The Lasts multiply to give 6 so 2×3 or 1×6.

You ignore + and – signs at this stage.

Step 2 Use FOIL to find the Outsides and Insides in each case:

$(x \quad 2)(x \quad 3)$ $3x$ and $2x$ Outsides: $x \times 3$ Insides: $2 \times x$

$(x \quad 1)(x \quad 6)$ $6x$ and x Outsides: $x \times 6$ Insides: $1 \times x$

Step 3 Attempt to get the middle term $(+5x)$ from the Outsides and Insides:

$(x \quad 2)(x \quad 3)$ $3x$ and $2x$ $+3x+2x$ Two + signs

$(x \quad 1)(x \quad 6)$ $6x$ and x $+6x-x$ One + and one – sign

Step 4 Fill in the +/– signs and check by expanding out using FOIL:

$(x+2)(x+3)=x^2+3x+2x+6$ not correct as -6 is required

$(x-1)(x+6)=x^2+6x-x-6$ correct as this gives x^2+5x-6

So $x^2+5x-6=(x-1)(x+6)$

Example Factorise $a^2-10a+16$

Working:

Possible Firsts and Lasts	Outsides	Insides	Combine to give $-10a$?
$(a \quad 4)(a \quad 4)$	$4a$	$4a$	not possible
$(a \quad 2)(a \quad 8)$	$8a$	$2a$	$-8a-2a$ gives $-10a$
$(a \quad 1)(a \quad 16)$	$16a$	a	not possible

Check $(a-2)(a-8)=a^2-8a-2a+16=a^2-10a+16$ ✓

More factorising of quadratic expressions

Example

To factorise $5x^2 - 29x - 6$ you must be careful to list ***all*** the possibilities. Since $5x$ and x are not identical Firsts then 'swapping' the Lasts creates different possibilities.

Possible Firsts and Lasts	Outsides	Insides	Combine to give $-29x$?
$(5x \quad 2)(x \quad 3)$	$15x$	$2x$	not possible
$(5x \quad 3)(x \quad 2)$	$10x$	$3x$	not possible
$(5x \quad 1)(x \quad 6)$	$30x$	x	$-30x + x = -29x$
$(5x \quad 6)(x \quad 1)$	$5x$	$6x$	not possible

Check: $(5x+1)(x-6) = 5x^2 - 30x + x - 6 = 5x^2 - 29x - 6$ ✓

So $5x^2 - 29x - 6 = (5x+1)(x-6)$

TOP TIP

Always take out any common factor first, e.g.
$2a^2 - 2b^2 = 2(a^2 - b^2)$
$= 2(a-b)(a+b)$

Difference of two squares

Choose a pair of square numbers, subtract the smaller from the larger then factorise the result:

49	–	25	=	24	
(7^2)		(5^2)		(2×12)	
81	–	16	=	65	
(9^2)		(4^2)		(5×13)	

A pattern can be spotted:

$a^2 - b^2$
$= (a-b)(a+b)$

Square numbers

1 (1^2), 4 (2^2), 9 (3^2), 16 (4^2), 25 (5^2), 36 (6^2), 49 (7^2), 64 (8^2), 81 (9^2), 100 (10^2)

Check this pattern for more differences of two of the squares above.

Example

Factorise:

$16x^2 - 1$

Solution

$16x^2 - 1 = (4x-1)(4x+1)$ (This is $(4x)^2 - 1^2$)

Completing the square

Notice this pattern in these square expressions:

$x^2 + 4x + 4 = (x + 2)^2$ (halve) $\quad x^2 - 2x + 1 = (x - 1)^2$ (halve)

You can use this pattern to complete expressions to make them squares:

$x^2 - 6x$ becomes $(x-3)^2 = (x-3)(x-3) = x^2 - 6x + 9$

$x^2 + 10x$ becomes $(x+5)^2 = (x+5)(x+5) = x^2 + 10x + 25$

But notice that 9 and 25 have been added to the original expressions.
By now subtracting 9 and 25 the original expressions will have been unaltered:

$x^2 - 6x = (x-3)^2 - 9$ and $x^2 + 10x = (x+5)^2 - 25$

Writing a quadratic expression in the form $(x+a)^2 + b$ is called 'completing the square'.

Example

Express in the form $(x+a)^2 + b$

$x^2 + 4x + 1$

Solution

$$\underbrace{x^2 + 4x + 1 = (x+2)(x+2)}_{x^2+4x+4} \underbrace{- 4}_{\text{to remove the unwanted } +4} + 1 = (x+2)^2 - 3$$

TOP TIP

You will use 'completing the square' to help you graph the values of quadratic expressions in Chapter 2.

Factorising

1. Factorise fully:

(a) $5xy + 15y^2$ (b) $7f^2g^2 - fg$
(c) $2pq^2 + 14pq - 7p^2$ (d) $rs^3 - 3rs + 6s^2$
(e) $t^2 + 8t + 12$ (f) $r^2 - 11r + 10$
(g) $y^2 + 6y + 5$ (h) $p^2 - 6p + 8$
(i) $24 - 11s + s^2$ (j) $w^2 + 2w - 15$
(k) $v^2 + 3v - 4$ (l) $15 + 2w - w^2$
(m) $z^2 + 12z - 13$ (n) $-x^2 + 4x - 3$
(o) $25 - 10f + f^2$ (p) $a^2 - 6a - 16$
(q) $x^2 - 8x - 9$ (r) $35 - 2q - q^2$
(s) $b^2 - b - 20$ (t) $h^2 + 14h + 49$
(u) $-1 + 2k - k^2$ (v) $3y^2 + 8y + 4$
(w) $2x^2 + 9x + 4$ (x) $6m^2 + 13m + 5$
(y) $10d^2 - 11d - 6$ (z) $9p^2 + 18p - 16$

2. Factorise fully:

(a) $x^2 - 25$ (b) $a^2 - 1$
(c) $a^2 - 100$ (d) $4p^2 - 9$
(e) $64p^2 - 121$ (f) $36 - 25u^2$
(g) $x^2 - 16y^2$ (h) $49t^2 - 144s^2$
(i) $f^2 - 900g^2$ (j) $5x^2 - 500$
(k) $3w^2 - 243$ (l) $10v^2 - 40$
(m) $12p^2 - 3$ (n) $20 - 45s^2$
(o) $7y^2 - 28z^2$ (p) $27a^2 - 48b^2$

Adding algebraic fractions

Your aim is to get the denominators the same, i.e. to make a **common denominator**.

Numerical

$$\frac{2}{5}+\frac{1}{3}=\frac{2\times3}{5\times3}+\frac{1\times5}{3\times5}=\frac{6}{15}+\frac{5}{15}$$

Fifths and thirds

different denominators

now both fifteenths

common denominators

$$\frac{6}{15}+\frac{5}{15}=\frac{6+5}{15}=\frac{11}{15}$$

you can now add the numerators.

Algebraic

$$\frac{a}{b}+\frac{c}{d}=\frac{a\times d}{b\times d}+\frac{b\times c}{b\times d}=\frac{ad}{bd}+\frac{bc}{bd}$$

different denominators

common denominators

$$\frac{ad}{bd}+\frac{bc}{bd}=\frac{ad+bc}{bd}$$

you can now add the numerators.

Examples

Express these as a single fraction in its simplest form: 1. $\frac{2}{m}+\frac{1}{n}$ 2. $\frac{5}{x}+\frac{2}{x-1}$

Solutions

1. $\frac{2}{m}+\frac{1}{n}=\frac{2\times n}{m\times n}+\frac{m\times1}{m\times n}=\frac{2n}{mn}+\frac{m}{mn}=\frac{2n+m}{mn}$
2. $\frac{5}{x}+\frac{2}{x-1}=\frac{5(x-1)}{x(x-1)}+\frac{2x}{x(x-1)}=\frac{5(x-1)+2x}{x(x-1)}=\frac{5x-5+2x}{x(x-1)}=\frac{7x-5}{x(x-1)}$

Finding the lowest common denominator

When making the denominators the same look closely at the factors of each denominator.

$\frac{2}{a}+\frac{1}{3a^2}$ Here the common denominator will be $3\times a\times a$ so a should be multiplied by $3a$

$$=\frac{2\times3a}{a\times3a}+\frac{1}{3a^2}=\frac{6a}{3a^2}+\frac{1}{3a^2}=\frac{6a+1}{3a^2}$$

$\frac{3}{2ab}+\frac{2}{3a^2b}$ Here the common denominator will be $2\times3\times a\times a\times b=6a^2b$. $2ab$ is multiplied by $3a$ and $3a^2b$ is multiplied by 2.

$$=\frac{3\times3a}{2ab\times3a}+\frac{2\times2}{3a^2b\times2}=\frac{9a}{6a^2b}+\frac{4}{6a^2b}=\frac{9a+4}{6a^2b}$$

Top Tip

Always check to see if your final answer can be cancelled down.

Examples

Express these as a single fraction:

1. $\frac{2}{3y^2}+\frac{1}{2y}$ 2. $\frac{5}{m^2n}+\frac{2}{mn^2}$

Solutions

1. $\frac{2}{3y^2}+\frac{1}{2y}=\frac{2\times2}{3y^2\times2}+\frac{1\times3y}{2y\times3y}=\frac{4}{6y^2}+\frac{3y}{6y^2}=\frac{4+3y}{6y^2}$
2. $\frac{5}{m^2n}+\frac{2}{mn^2}=\frac{5\times n}{m^2n\times n}+\frac{2\times m}{mn^2\times m}=\frac{5n}{m^2n^2}+\frac{2m}{m^2n^2}=\frac{5n+2m}{m^2n^2}$

Subtracting algebraic fractions

When you have a common denominator subtract the numerators:

$\frac{a}{b} - \frac{c}{d} = \frac{a \times d}{b \times d} - \frac{b \times c}{b \times d} = \frac{ad}{bd} - \frac{bc}{bd} = \frac{ad - bc}{bd}$

TOP TIP

Any simple term like a, b, $2x$, y^2 can be written as a fraction:

$\frac{a}{1}, \frac{b}{1}, \frac{2x}{1}, \frac{y^2}{1}$.

Example

Express $\frac{x}{x-1} - \frac{x}{x+1}$ as a single fraction in its simplest form.

Solution

$\frac{x}{x-1} - \frac{x}{x+1} = \frac{x(x+1)}{(x-1)(x+1)} - \frac{x(x-1)}{(x-1)(x+1)} = \frac{x(x+1) - x(x-1)}{(x-1)(x+1)} = \frac{x^2 + x - x^2 + x}{(x-1)(x+1)} = \frac{2x}{x^2-1}$

Some special cases

To simplify $1 - \frac{1}{a}$ rewrite 1 as $\frac{a}{a}$: $\frac{a}{a} - \frac{1}{a} = \frac{a-1}{a}$

(suppose $a = 5$ then $1 - \frac{1}{a} = 1 - \frac{1}{5} = \frac{4}{5}$ and $\frac{a-1}{a} = \frac{5-1}{5} = \frac{4}{5}$).

To simplify $a + \frac{b}{c}$ remember $a = \frac{a}{1}$ so you get $\frac{a \times c}{1 \times c} + \frac{b}{c} = \frac{ac+b}{c}$.

A further example: $a + 1 + \frac{1}{a} = \frac{a^2}{a} + \frac{a}{a} + \frac{1}{a} = \frac{a^2+a+1}{a}$

An application

The resistance (R) of two resistors wired in parallel is given by: $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$. Let's add the fractions: $\frac{1}{R_1} + \frac{1}{R_2} = \frac{1 \times R_2}{R_1 \times R_2} + \frac{R_1 \times 1}{R_1 \times R_2} = \frac{R_2}{R_1 R_2} + \frac{R_1}{R_1 R_2} = \frac{R_1 + R_2}{R_1 R_2}$. So $\frac{1}{R} = \frac{R_1 + R_2}{R_1 R_2}$.

Now 'invert' both fractions: $\frac{R}{1} = \frac{R_1 R_2}{R_1 + R_2}$ or $R = \frac{R_1 R_2}{R_1 + R_2}$, a more useful form of the formula.

Preparation for Higher

Algebraic Fractions

1. Express each of the following as a single fraction and simplify where possible:

(a) $\frac{e}{5}+\frac{2e}{5}$ (b) $\frac{5m}{7}-\frac{2m}{7}$ (c) $\frac{5r}{12}-\frac{11r}{12}$

(d) $\frac{a}{3}+\frac{b}{3}$ (e) $\frac{3p}{7}-\frac{p}{7}$ (f) $\frac{13}{v}-\frac{2}{v}$

(g) $\frac{8}{w}-\frac{9}{w}$ (h) $\frac{2a}{u}+\frac{3a}{u}$ (i) $\frac{a}{x}-\frac{b}{x}$

2. Express each of the following as a single fraction and simplify where possible:

(a) $\frac{5x}{6}-\frac{2x}{3}$ (b) $\frac{n}{5}+\frac{3n}{20}$ (c) $\frac{7u}{8}-\frac{5u}{16}$

(d) $\frac{p}{4}-\frac{q}{2}$ (e) $\frac{3x}{25}-\frac{y}{5}$ (f) $\frac{3a}{22}+\frac{3b}{2}$

(g) $\frac{1}{a}+\frac{3}{2a}$ (h) $\frac{5}{3b}-\frac{1}{b}$ (i) $\frac{8}{b}-\frac{7}{d}$

(j) $\frac{1}{13p}-\frac{5}{39p}$ (k) $\frac{3}{2x}+\frac{1}{2}$ (l) $\frac{3}{5}-\frac{2}{5a}$

(m) $\frac{a}{2}-\frac{a}{9}$ (n) $\frac{2t}{3}+\frac{t}{8}$ (o) $\frac{3h}{5}-\frac{2h}{7}$

(p) $\frac{u}{3}+\frac{v}{2}$ (q) $\frac{5h}{7}-\frac{4k}{9}$ (r) $\frac{2c}{3}+\frac{3d}{10}$

(s) $\frac{4}{5m}-\frac{1}{2m}$ (t) $\frac{3}{4r}+\frac{5}{6r}$ (u) $\frac{2}{9t}-\frac{3}{4t}$

3. Express each of the following as a single fraction and simplify where possible:

(a) $\frac{a}{2}+\frac{3}{b}$ (b) $\frac{x}{5}-\frac{4}{y}$ (c) $\frac{7}{r}+\frac{s}{9}$

(d) $\frac{8}{u}-\frac{t}{3}$ (e) $\frac{3m}{5}-\frac{7}{2n}$ (f) $\frac{6}{7v}-\frac{2w}{3}$

(g) $\frac{c}{p}+\frac{d}{q}$ (h) $\frac{9}{x}+\frac{5}{y}$ (i) $\frac{8}{b}-\frac{7}{d}$

(j) $\frac{p}{e}-\frac{3}{f}$ (k) $\frac{2a}{7}+\frac{9}{3b}$ (l) $\frac{5}{9}-\frac{x}{2f}$

(m) $\frac{x+1}{2}+\frac{x-2}{3}$ (n) $\frac{a-5}{4}-\frac{a+3}{5}$ (o) $\frac{q+4}{3}+\frac{q-6}{7}$

(p) $\frac{w-7}{8}-\frac{w-1}{9}$ (q) $\frac{t+12}{6}+\frac{t}{10}$ (r) $\frac{2x+3}{5}+\frac{7-x}{9}$

Preparation for Higher

Complex Algebraic Fractions

1. Express as a single fraction:

(a) $\frac{2}{a+1}+\frac{3}{a}$ (b) $\frac{4}{w+3}-\frac{7}{w}$ (c) $\frac{6}{e-5}+\frac{9}{e}$

(d) $\frac{5}{m}-\frac{8}{m+2}$ (e) $\frac{1}{r}+\frac{9}{r-7}$ (f) $\frac{7}{v}-\frac{3}{v-9}$

(g) $\frac{3}{b+1}+\frac{2}{b+3}$ (h) $\frac{4}{n-5}+\frac{6}{n+2}$ (i) $\frac{5}{s+4}-\frac{8}{s+7}$

(j) $\frac{7}{t-2}-\frac{9}{t+5}$ (k) $\frac{5}{x-1}+\frac{1}{x-7}$ (l) $\frac{9}{y-8}-\frac{6}{y-5}$

2. Express as a single fraction:

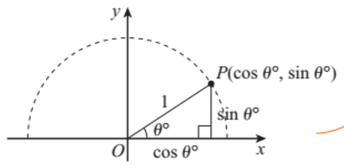
(a) $\frac{2}{a^2-1}+\frac{1}{a+1}$ (b) $\frac{1}{x^2-1}-\frac{1}{x-1}$

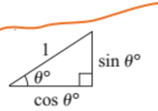
(c) $\frac{1}{b+3}+\frac{3}{b^2+4b+3}$ (d) $\frac{2}{w^2-2w+1}+\frac{3}{w-1}$

(e) $\frac{1}{P-2}-\frac{3}{P^2+P-6}$ (f) $\frac{2}{x+2}-\frac{5}{x^2-3x-10}$

(g) $\frac{1}{c^2-2c+1}+\frac{1}{c^2-1}$ (h) $\frac{m+4}{m^2-9}-\frac{1}{m-3}$

Some trig formulae

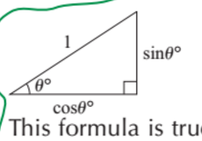




Use SOHCAHTOA to get:

$$\tan\theta° = \frac{\sin\theta°}{\cos\theta°}$$

This formula is true for all values of $\theta°$ (other than 90°, 270° etc.).



Use Pythagoras' Theorem to get: $(\sin\theta°)^2 + (\cos\theta°)^2 = 1^2$

so $\sin^2\theta° + \cos^2\theta° = 1$.

This formula is true for all values of $\theta°$.

TOP TIP

$\sin x° \times \sin x°$ is written $(\sin x°)^2$ or $\sin^2 x°$ but not $\sin x°^2$.

Rearrangements of these formulae are possible:

$\dfrac{\sin\theta°}{\cos\theta°} = \tan\theta°$ so $\sin\theta° = \cos\theta° \tan\theta°$

(after multiplying both sides by $\cos\theta°$)

so $\dfrac{\sin\theta°}{\tan\theta°} = \cos\theta°$ (after dividing both sides by $\tan\theta°$)

$\sin^2\theta° + \cos^2\theta° = 1$ so $\sin^2\theta° = 1 - \cos^2\theta°$ or $\cos^2\theta° = 1 - \sin^2\theta°$

Example

Show that $\dfrac{\sin^2 A}{1-\sin^2 A} = \tan^2 A$

Solution

$$\frac{\sin^2 A}{1-\sin^2 A} = \frac{\sin^2 A}{\cos^2 A} \quad \text{(using } 1-\sin^2 A = \cos^2 A\text{)}$$

$$= \frac{\sin A}{\cos A} \times \frac{\sin A}{\cos A}$$

$$= \tan A \times \tan A = \tan^2 A \text{ as required.}$$

1. Prove the following identities:

   (a) $(x+1)^2 = (x-1)^2 + 4x$
   (b) $(x-y)^2 = (y-x)^2$
   (c) $(p+5)(p+1) = -8 + (p+3)^2$
   (d) $4(a^2-4) = (2a-4)^2 + 16(a-2)$
   (e) $(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$
   (f) $(x+2)^3 = x^3 + 2(3x^2+6x+4)$
   (g) $(6x-y)^2 - (4x-y)(9x-y) = xy$
   (h) $(px-qy)^2 + (qx+py)^2 = (p^2+q^2)(x^2+y^2)$

2. Prove the identities:

   (a) $5\cos^2 x + 5\sin^2 x = 5$
   (b) $2\cos^2 x - 1 = 1 - 2\sin^2 x$
   (c) $(\cos P + \sin P)^2 = 2\sin P\cos P + 1$
   (d) $2\sin a\cos a + (\cos a + \sin a)^2 = 1$
   (e) $(\cos y + \sin y)(\cos y - \sin y) = 1 - 2\sin^2 y$
   (f) $\cos a \tan a = \sin a$
   (g) $\cos^2 q \tan^2 q = 1 - \cos^2 q$
   (h) $\dfrac{\cos a}{\sin a} - \dfrac{\sin a}{\cos a} = \dfrac{2\cos^2 a - 1}{\sin a\cos a}$