Functions & curve sketching

What is a function?

A function describes the relationship between two sets of quantities where one set depends on the other set.

The letters f, g and h are usually used for the names of functions.

Think of a function as:

an input → a calculation → an output

e.g. distance travelled → e.g. fuel used

e.g. radius of circle → e.g. area of circle

Examples

Amount of fuel used is a function of the distance travelled

Area of a circle is a function of the radius of the circle

Function notation

Here is an example of function notation and its meaning:

$$f(x) = x^2$$

- The name of the function is f
- The input variable is x
- The formula is x^2

You may evaluate this function for a particular value of x:

$f(5) = 5^2 = 25$.

Every occurrence of x is replaced by 5.
The value of f when $x = 5$ is 25.

An alternative notation is:

$f : x \to x^2$

$f : 5 \to 5^2$

The input is 5 and the output is 25.

Example

Given that $g(n) = n^2 - 4n$

evaluate $g(-2)$

Solution

$g(-2) = (-2)^2 - 4 \times (-2) = 4 + 8 = 12$

1.	If	$f(x) = 3x - 4$	Evaluate:	(a)	$f(2)$	(b)	$f(-1)$
2.	If	$f(x) = x^2 - 1$	Evaluate:	(a)	$f(4)$	(b)	$f(-2)$
3.	If	$f(x) = 2x^3 + 3$	Evaluate:	(a)	$f(3)$	(b)	$f(-1)$
4.	If	$f(x) = 3x^2$	Evaluate:	(a)	$f(5)$	(b)	$f(-4)$
5.	If	$f(x) = 3x^2 - 1$	Evaluate:	(a)	$f(4)$	(b)	$f(-2)$
6.	If	$f(x) = 7 - x$	Evaluate:	(a)	$f(3)$	(b)	$f(-7)$
7.	If	$f(x) = 5 - x^2$	Evaluate:	(a)	$f(2)$	(b)	$f(-3)$
8.	If	$f(x) = -x^3$	Evaluate:	(a)	$f(1)$	(b)	$f(-4)$
9.	If	$f(x) = 4 + x^2$	Evaluate:	(a)	$f(5)$	(b)	$f(-3)$
10.	If	$f(x) = 3 + 2x - x^3$	Evaluate:	(a)	$f(2)$	(b)	$f(-1)$

Quadratic Functions

The parabola

A graph showing the values of x^2 for all values of x can be built up from a few particular values.

x	−3	−2	−1	0	1	2	3
$y = x^2$	9	4	1	0	1	4	9

(−3, 9) (−2, 4) (−1, 1) (0, 0) (1, 1) (2, 4) (3, 9)

Notes

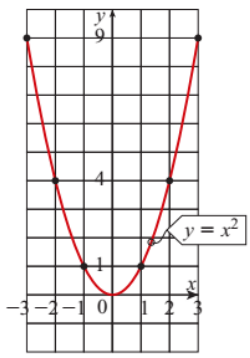
(1) This type of graph shape is called a **parabola**.

(2) The graph has symmetry, with the y-axis (the line $x = 0$) being the axis of symmetry.

(3) The graph has a minimum turning point at the origin (0, 0). This means that x^2 has a minimum value of 0 when $x = 0$.

(4) The equation $y = 0$ or $x^2 = 0$ has one **solution (root)**, namely $x = 0$.

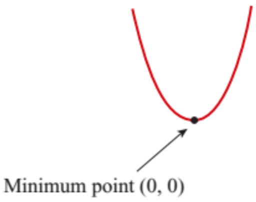
The graph $y = x^2$



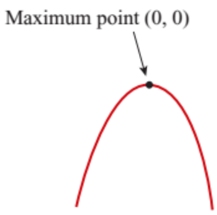
The graph $y = kx^2$

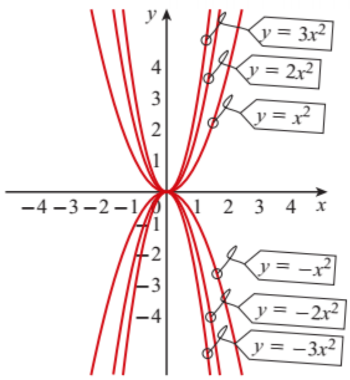
TOP TIP
In your exam $y = kx^2$ graphs will only be considered with k as an integer.

For $k > 0$ (positive) the graph is concave upwards:



For $k < 0$ (negative) the graph is concave downwards:

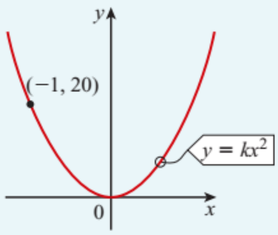




Note: The value of k affects the steepness but not the turning point of the curve.

Example

Use the information in the diagram to calculate the value of k.



Solution

$(-1, 20)$ lies on the curve so $x = -1$ and $y = 20$ satisfy the equation $y = kx^2$

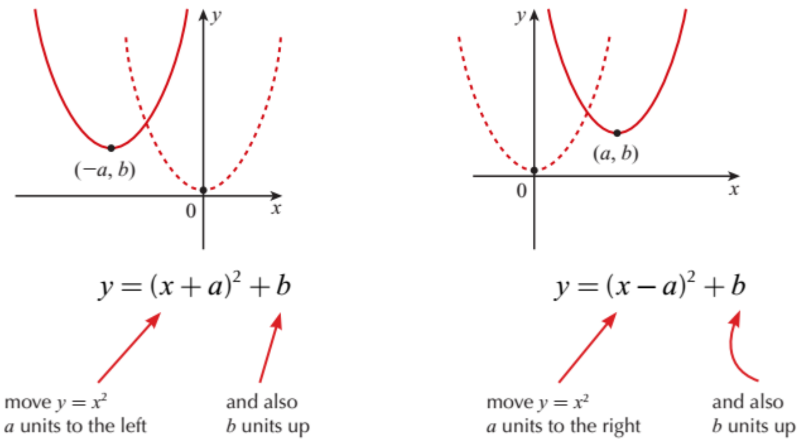
This gives: $20 = k \times (-1)^2$

so $20 = k \times 1$

and therefore $k = 20$.

The graphs $y = (x + a)^2 + b$ and $y = (x - a)^2 + b$

The graph $y = x^2$ (see previous page) is moved to get:



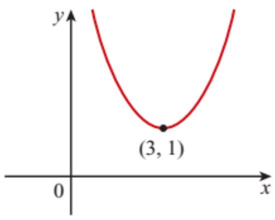
Note: when given a quadratic expression like $x^2 + 4x + 7$ you can 'complete the square' (see page 28) to get $(x + 2)^2 + 3$ and 'read off' the minimum turning point (–2, 3).

Example

Give the coordinates of the minimum turning point of the graph $y = x^2 - 6x + 10$.

Solution

$y = x^2 - 6x + 10 = (x - 3)^2 + 1$ so $y = x^2$ is moved 3 units to the right and 1 unit up:



The minimum turning point is (3, 1).

Quadratic graphs and factors

To find the x-axis intercepts you set $y = 0$. For example, the graph $y = 2x^2 - x - 3$ crosses the x-axis when $y = 2x^2 - x - 3 = 0$.

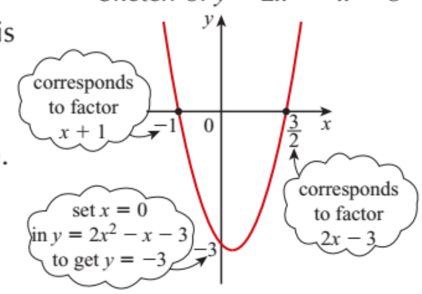
Solving this quadratic equation gives:

$(2x-3)(x+1) = 0$ so $x = \frac{3}{2}$ and $x = -1$ (see page 67).

The 'x^2-term' is positive $(2x^2)$ and so the graph is 'concave upwards' (see page 70).

The y-axis intercept is found when you set $x = 0$ in the equation.

Sketch of $y = 2x^2 - x - 3$



Sketching quadratic graphs – hints

- $y = ax^2 + bx + c$ If $a > 0$ (positive) the graph is 'concave upwards'.
  If $a < 0$ (negative) the graph is 'concave downwards'.
- Where does it cross the y- axis? → set $x = 0$ to find the value of y.

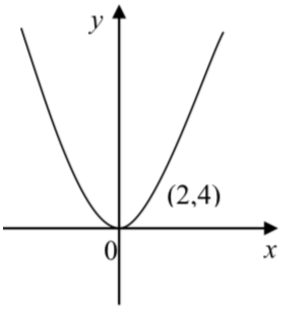
  Where does it cross the x- axis? → set $y = 0$ and solve the resulting equation.
- Complete the square to get $y = (x \pm a)^2 + b$ (see page 71) to find the turning point.
- Plot a few points: choose a value for x and calculate y.

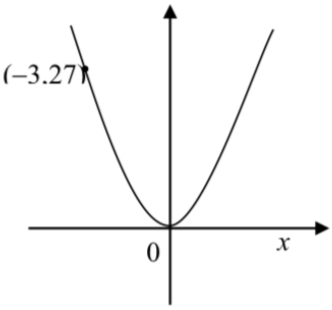
TOP TIP

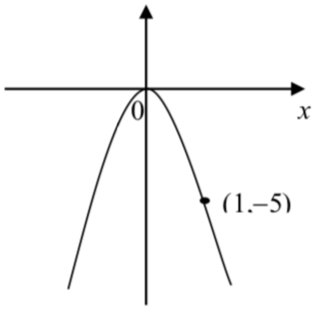
Calculating the **discriminant** $b^2 - 4ac$ tells you a lot about the graph – see page 74.

Worksheet

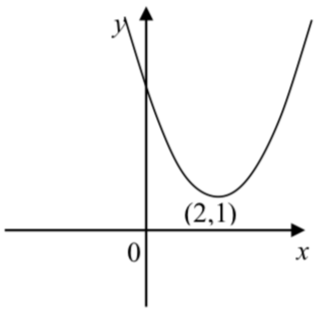
1. Write down the equation of the graphs shown below, which have the form $y = ax^2$.
   (Diagrams are not drawn to scale)

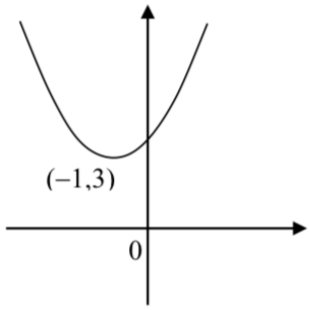


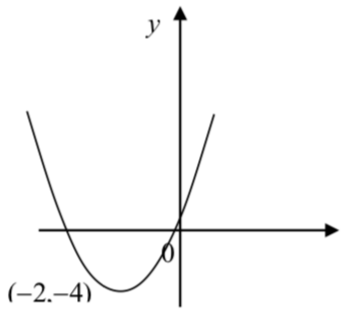




2. Write down the equation of the graphs shown below, which have the form
   $y = (x + a)^2 + b$. (Diagrams are not drawn to scale)







3. Sketch the following quadratic functions

a. $y = x(x - 5)$ **b**. $y = x(x + 7)$ **c**. $y = (a - 4)(a - 2)$

d. $y = (w + 1)(w + 2)$ **e**. $y = (x + 3)(x - 1)$ **f**. $y = (x - 4)^2 + 1$

3. For each of the equations below, write down
   i. the turning point
   ii. its nature
   iii. the equation of the axis of symmetry

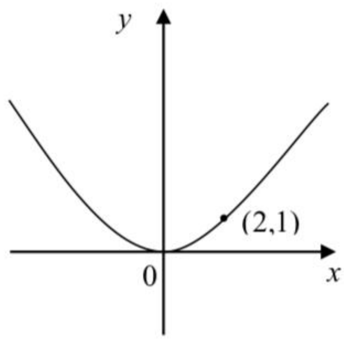
a. $y = (x - 3)^2 + 1$ b. $y = (x - 3)^2 - 4$ c. $y = (x + 1)^2 - 7$

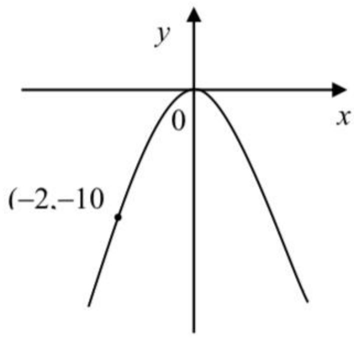
d. $y = (x + 2)^2 + 3$ e. $y = -(x - 1)^2 + 5$

4. For each equation, draw a suitable sketch and find the roots.

a. $x^2 - 4x = 0$ b. $x^2 + 8x + 12 = 0$ c. $x^2 - 5x + 4 = 0$

5. Write down the equation of the graphs in the form $y = ax^2$

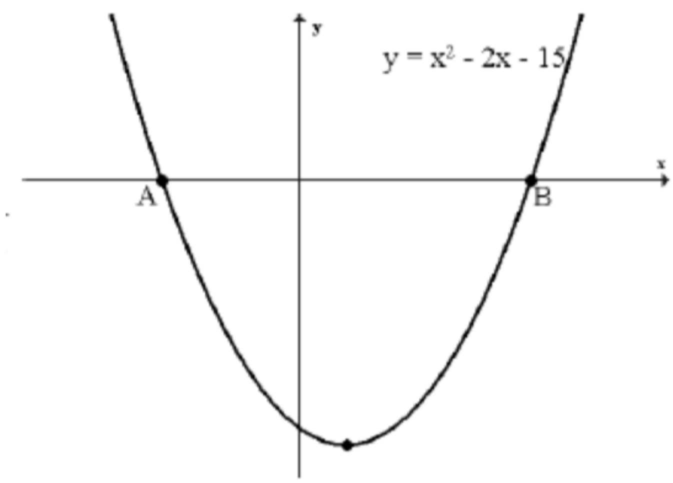




6. For each equation, draw a suitable sketch and find the roots.

a) $x^2 - 4x = 0$ b) $x^2 + 6x + 8 = 0$ c) $x^2 - 7x + 10 = 0$

7. The diagram shows part of the graph of $y = x^2 - 2x - 15$
   a) Find the coordinates of A and B.
      Find the minimum turning point on the curve.



Solving trig equations using the quadrant diagram

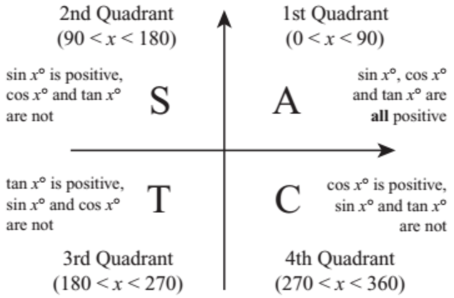
Step 1 Rearrange the equation (if possible!) to the form:

sin(angle) = number

or cos(angle) = number

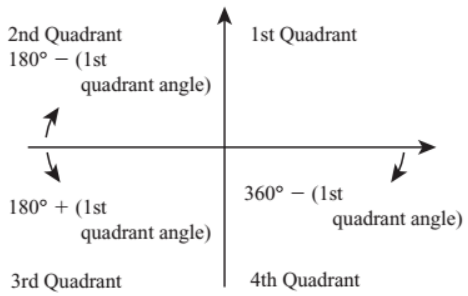
or tan(angle) = number

Step 2 Is 'number' positive or negative? Use the answer to this question and the quadrant diagram below to find out which quadrant 'angle' is in.



Step 3 Use $\sin^{-1}$ or $\cos^{-1}$ or $\tan^{-1}$ on your calculator and the **positive** value of 'number' to find the 1st quadrant angle.

Step 4 The information from Step 2 and Step 3 is used to calculate the solutions to the equation. Use this diagram:



Step 5 Check your solutions by substituting into the equation to see if they work!

Example

Solve $5\sin x° + 1 = 0$, $0 \leq x \leq 360$

Solution

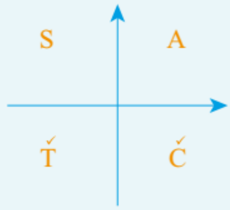
Step 1 $5\sin x° = -1$

$\sin x° = -\frac{1}{5}$

$\sin x° = -0 \cdot 2$

Step 2 $-0 \cdot 2$ is negative.

Using the diagram:



$\sin x°$ is negative in the 3rd quadrant and the 4th quadrant.

Step 3 $\sin^{-1}(0 \cdot 2) = 11 \cdot 5°$ (This is rounded to 1 decimal place.)

using a **positive** value

Step 4 The 3rd quadrant angle is $180° + 11 \cdot 5° = 191 \cdot 5°$.

The 4th quadrant angle is $360° - 11 \cdot 5° = 348 \cdot 5°$.

Step 5 Checking on the calculator:

$5\sin 191 \cdot 5° + 1 = 0 \cdot 0031....$

$5\sin 348 \cdot 5° + 1 = 0 \cdot 0031....$

These round to 0!

Solve the following: $(0 \leq x \leq 360)$

1. $2sinx - 1 = 0$
2. $2cosx - \sqrt{3} = 0$
3. $5tanx - 1 = 2$
4. $6sinx + 2 = 3$
5. $3cosx + 1 = 3$
6. $2tanx + 11 = 20$
7. $5sinx - 1 = -3$
8. $4cosx + 7 = 5$
9. $2tanx + 3 = 1$
10. $20sinx + 17 = 25$